LESSON 6.2

Evaluating Polynomials

LEARNING GOALS:

1) Determine if a function is a polynomial function or not.
2) Identify the degree, type, leading coefficient, constant term, and standard form of a polynomial function.
3) Evaluate a polynomial function using Direct Substitution.
4) Evaluate a polynomial function using Synthetic Substitution.

Common Core State Standards
HSF-IF.A.2

LESSON 6.2 - Evaluating Polynomial Functions

FUNCTIONS WE ARE ALREADY FAMILIAR WITH:

Degree	Type	Standard Form	Example
0	Constant	$f(x) = a_0$	$y = 6$
1	Linear	$f(x) = a_1x + a_0$	$y = 7x + 6$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$y = 2x^2 + 7x + 6$

NEW COMMON POLYNOMIAL FUNCTIONS:

Degree	Type	Standard Form	Example
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$y = 4x^3 + 2x^2 + 7x + 6$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$y = 2x^4 + 4x^3 + 2x^2 + 7x + 6$
5	Quintic	$f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$y = x^5 + 2x^4 + 4x^3 + 2x^2 + 7x + 6$
and so on...			

A polynomial function is a function of the form

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$$

such that the following properties are true:

- $a_n \neq 0$ (the leading coefficient cannot be equal to 0. However, other coefficients may be equal to 0.)
- All exponents are whole numbers (they cannot be negative, nor can they be fractions/decimals).
- All coefficients are real and rational (they cannot be imaginary, nor can they be non-repeating, non-terminating decimals).

A polynomial function is a function of the form

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$$

- a_n is called the **LEADING COEFFICIENT**
- a_0 is called the **CONSTANT TERM**
- n is called the **DEGREE**
- A polynomial function is in **STANDARD FORM** if its terms are written in descending order of exponents from left to right.

Determine if each function is a polynomial function or not.

A) $f(x) = 2x^3 + 3x^2 - 7x + 10$ YES

B) $f(x) = \sqrt{7}x^2 - 5x + \pi$ NO

C) $f(x) = 3x^5 + 9x^2 - x$ YES

D) $f(x) = 5x^{-4} + 8x^3 - x^2 + 6x - 11$ NO

E) $f(x) = 6^x - 4x^5 + 7$ NO

State the degree, type, and leading coefficient of each polynomial function.

A) $f(x) = 2x^3 - 9x^2 + 8x - 1$

DEGREE: 3 TYPE: cubic LC: 2

B) $f(x) = -3x - 7$

DEGREE: 1 TYPE: linear LC: -3

C) $f(x) = x^2 - 5x + 6$

DEGREE: 2 TYPE: quadratic LC: 1

6.2 NOTES - Evaluating Polynomial Functions

State the degree, type, and leading coefficient of each polynomial function.

D) $f(x) = 4$

DEGREE: 0 TYPE: constant LC: 4

E) $f(x) = 10x^5 + 7x^2 - 4x + 3$

DEGREE: 5 TYPE: quintic LC: 10

F) $f(x) = -8x^4 + 11$

DEGREE: 4 TYPE: quartic LC: -8

Rewrite the polynomial function in Standard Form

A) $f(x) = 6x + 7x^3 - 5 + x^2$

$f(x) = 7x^3 + x^2 + 6x - 5$

B) $f(x) = 8 - x^5 + 4x^3 + 10x^2$

$f(x) = -x^5 + 4x^3 + 10x^2 + 8$

C) $f(x) = x - 2x^3 + 9x^4$

$f(x) = 9x^4 - 2x^3 + x$

EVALUATING A POLYNOMIAL FUNCTION FOR A GIVEN VALUE OF x

- To **EVALUATE A FUNCTION** means to calculate the value of the function at a certain number.
- In other words, plug in a number for x, and solve.
- There are two methods for evaluating functions:

Method 1: Direct Substitution

Plug the given value of x into the function everywhere you see an x, then simplify.

Method 2: Synthetic Substitution

An easier way to evaluate larger functions.

Use Direct Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^4 - 8x^2 + 5x - 7$ for $x = 3$

$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7$

$f(3) = 2(81) - 8(9) + 15 - 7$

$f(3) = 162 - 72 + 15 - 7$

$f(3) = 98$

METHOD 2: Synthetic Substitution

STEP 1: Draw a large/long L

STEP 2: Write the coefficients along the top

NOTE: If there are any terms missing from the polynomial, write a 0 in that term's place.

STEP 3: Write the given value of x on the left outside

STEP 4: Drop the first coefficient down and write it in the first spot under the L.

STEP 5: Multiply the given value of x by that first coefficient. Write the answer inside the L under the second coefficient.

STEP 6: Add the second coefficient and the answer from Step 5. Write the answer in line with those two numbers under the L.

STEP 7: Repeat this process until you run out of coefficients. The last number you write under the L (in line with the last coefficient) is the answer.

Use Synthetic Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^4 - 8x^2 + 5x - 7$ for $x = 3$

	2	0	-8	5	-7
3		6	18	30	105
	2	6	10	35	98

Use Direct Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^3 + 5x^2 + 4x + 8$ for $x = -2$

$f(-2) = 2(-2)^3 + 5(-2)^2 + 4(-2) + 8$

$f(-2) = 2(-8) + 5(4) - 8 + 8$

$f(-2) = -16 + 20 - 8 + 8$

$f(-2) = 4$

Use Synthetic Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^3 + 5x^2 + 4x + 8$ for $x = -2$

$$\begin{array}{r|rrrr} & 2 & 5 & 4 & 8 \\ -2 & & -4 & -2 & -4 \\ \hline & 2 & 1 & 2 & 4 \end{array}$$

Use Direct Substitution to evaluate the polynomial for the given value of x.

$f(x) = 5x^4 - 6x^3 - 7x^2$ for $x = 3$

$f(3) = 5(3)^4 - 6(3)^3 - 7(3)^2$

$f(3) = 5(81) - 6(27) - 7(9)$

$f(3) = 405 - 162 - 63$

$f(3) = 180$

Use Synthetic Substitution to evaluate the polynomial for the given value of x.

$f(x) = 5x^4 - 6x^3 - 7x^2$ for $x = 3$

$$\begin{array}{r|rrrrr} & 5 & -6 & -7 & 0 & 0 \\ 3 & & 15 & 27 & 60 & 180 \\ \hline & 5 & 9 & 20 & 60 & 180 \end{array}$$

Use Direct Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^3 - x^4 + 5x^2 - 4 - x$ for $x = 4$

$f(4) = 2(4)^3 - (4)^4 + 5(4)^2 - 4 - 4$

$f(4) = 2(64) - 256 + 5(16) - 4 - 4$

$f(4) = 128 - 256 + 80 - 4 - 4$

$f(4) = -56$

Use Synthetic Substitution to evaluate the polynomial for the given value of x.

$f(x) = 2x^3 - x^4 + 5x^2 - 4 - x$ for $x = 4$

$$\begin{array}{r|rrrrr} & -1 & 2 & 5 & -1 & -4 \\ 4 & & -4 & -8 & -12 & -52 \\ \hline & -1 & -2 & -3 & -13 & -56 \end{array}$$

Use Direct Substitution to evaluate the polynomial for the given value of x.

$$f(x) = x + \frac{1}{2}x^3 \quad \text{for} \quad x = 4$$

$f(4) = 4 + \frac{1}{2}(4)^3$

$f(4) = 4 + \frac{1}{2}(64)$

$f(4) = 4 + 32$

$f(4) = 36$

Use Synthetic Substitution to evaluate the polynomial for the given value of x.

$$f(x) = x + \frac{1}{2}x^3 \quad \text{for} \quad x = 4$$

$$\begin{array}{c|cccc} & \frac{1}{2} & 0 & 1 & 0 \\ 4 & & 2 & 8 & 36 \\ \hline & \frac{1}{2} & 2 & 9 & 36 \end{array}$$

HOMEWORK:

6.2 Worksheet - Evaluating Polynomials